Random Variation of Exchange Rates in the Equal Utility Exchange Model

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The exchange model of Stokman and Van Oosten (1994) assumes that actors, when exchanging, agree upon the exchange rate that provides equal utility gains to them; the equal gain solution (EG). Using an algorithm that orders exchange opportunities within but also across actors, exchanges opportunities are subsequently executed, deleted or updated, following a set of restrictions, from first to last (most to least beneficial) until no opportunities are left.

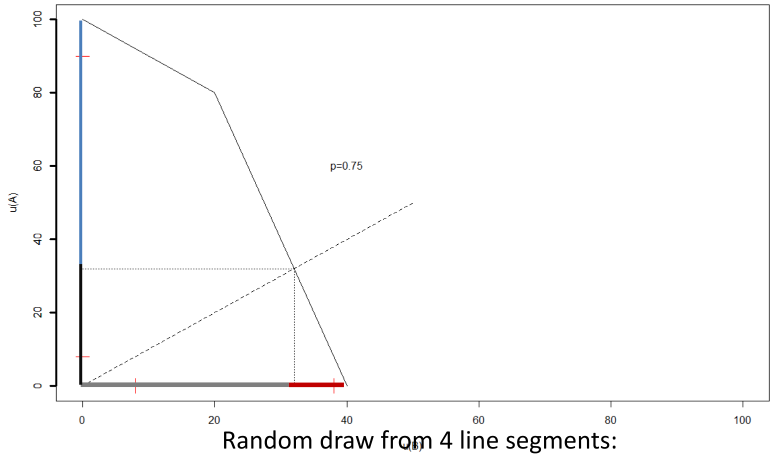
Although the outcome of the algorithm is not necessarily deterministic (e.g., if ties occur one of the tied opportunities is randomly selected for execution), the algorithm leaves no room for uncertainty in the outcomes of individual exchanges. However, using decision theoretic arguments it is reasonable to assume that outcomes of complex decision problems, particularly complex decisions such as collective decision making, are subject to a certain extent of 'decision error' or randomness (whatever its origin). The exchange model may also imperfectly predict the outcome of exchanges for other reasons. All these reasons together ask for an analysis allowing for an examination of the stability of the outcomes predicted by the Stokman and Van Oosten (1994) exchange model.

We incorporated randomness into the exchange model to examine the stability of the original predictions; is the prediction robust to random decision error or other random perturbations? The random exchange model uses the same algorithm as the Stokman and Van Oosten (1994) model, supplemented with a statistical model operating at the level of an individual exchange. It has one parameter p representing the extent of randomness, with $p = 0$ coinciding with the Stokman and Van Oosten (1994) and $p = 1$ implying a 'maximally random' decision outcome at the level of individual exchanges. We use an example to illustrate the statistical model of the random exchange model.

Figure 1 presents the two-dimensional payoff space of actors A and B in a hypothetical exchange relation, with x and y axis representing the utility gains of B and A, respectively, and with the space bounded to the right by the Pareto-optimal payoff frontier. The diagonal presents the outcomes yielding an equal utility gain, with the EG solution equal to where the diagonal crosses the frontier. This is also the solution of the random exchange model for $p = 0$. The basic idea of the random exchange model for $p > 0$ is simple; whatever the reason, the actual outcome on the frontier may be such that A (or B) profits more or less than in the EG, with 50% chance he profits more, and the value of p determines the extent to which he may profit more (or less).

Turning back to Figure 1, take the perspective of actor A and assume that $p = 0.75$. In the exchange model, $p = 0.75$ means that actor A can additionally gain (on top of what he gains in the EG solution) up to 75% of the difference between what he can maximally earn (A_{max}) and what he earns in the EG solution. Similarly, it also means he can maximally lose 75% of his gain, relative to what he gains in the EG. In the random exchange model, the randomly selected exchange rate of one exchange is determined by first randomly selecting which actor wins or loses (all with probability .25), and then randomly selecting the utility gain of that actor from the uniform distribution with as bounds his gain from the EG solution and the maximum gain or loss determined by p. This is illustrated for our example in Figure 2. In Figure 2, where it was randomly determined that actor A gains relative more than what he would obtain in the EG solution.

Figure 1: Random variation of gains: example

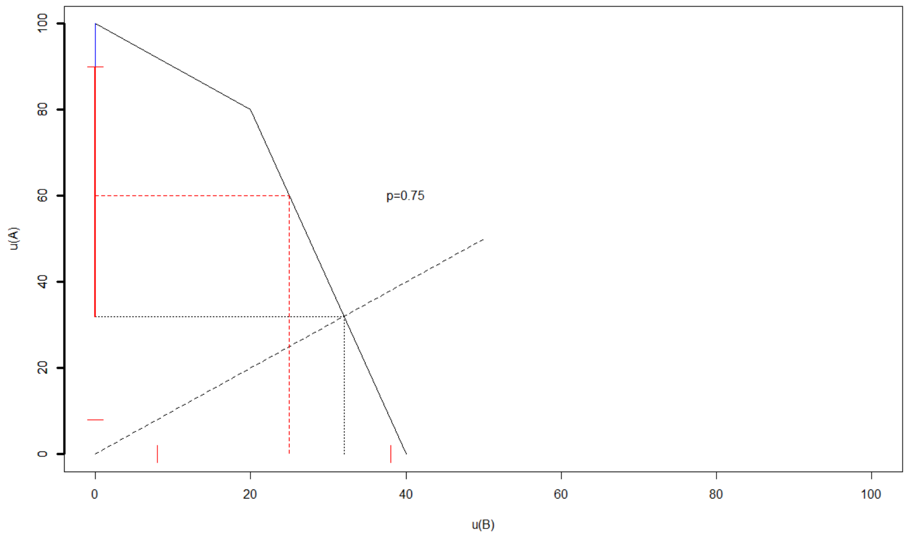


Random draw from 4 line segments:

1. Y-axis, *above* EG (below red bar)
2. Y-axis, *below* EG (above red bar)
3. X-axis, *left* of EG (right of red bar)
4. X-axis, *right* of EG (left of red bar)

Choice of *p* determines width of interval

Figure 2: Example of randomly selected exchange rate



Random utility for A is 60, implying utility of 25 for B

The researcher not only chooses *p* but also the number of iterations of the random exchange model, that is, the number of times the model is run of the dataset. As in each iteration of the model, other random numbers are drawn and therefore other exchanges may be carried out, the output of the model may be different in each iteration. The output of a run of the exchange model includes the average, standard deviation, and frequency distribution of:

- predicted decision outcome (at issue level)
- utility gain (at actor level)
- exchanges (at level of exchanges)

Table 1 gives an impression of the effects of the random component on the predicted outcome of one main issue at the COP Copenhagen study (Stokman 2015). The issue concerns the question whether COP Copenhagen would result in a new Treaty (position 0) vs in an extension of the earlier Kyoto Treaty (position 100). The Table shows the voting positions of three dominant actors of the eleven actor groups at the Copenhagen Climate conference, comparing p=0.0 with p=1.0.

Table 1: Summary of partial results of Copenhagen Treaty simulation for p=0.0 and p=1.0

p=0.0				**p=1.0**			
New Treaty	10	100		New Treaty	10	100	
Overview issue				Overview issue			
round	nbs avg	nbs var		round	nbs avg	nbs var	nbs var avg
rn-0	61.58	0.00		rn-0	61.58	0.00	1576.33
rn-9	56.38	1.82		rn-9	56.59	2.78	355.80
AVG Preference development NBS and all actors				*AVG Preference development NBS and all actors*			
actor	*salience*	*rnd-0*	*rnd-9*	*actor*	*rnd-0*	*rnd-9*	
China-India	0.9	100	97.97	China-India	100.00	97.08	
EU incl. Norway	0.4	50	59.44	EU incl. Norway	44.44	53.74	
USA	0.9	10	17.92	USA	0.00	9.78	
Standard deviation				*Standard deviation*			
actor		*rnd-0*	*rnd-9*	*actor*	*rnd-0*	*rnd-9*	
China-India		0	0.69	China-India	0.00	0.77	
EU incl. Norway		0	**9.44**	EU incl. Norway	0.00	**10.70**	
USA		0	2.61	USA	0.00	3.33	

References

Stokman, Frans N., and Reinier Van Oosten, 1994
The Exchange of Voting Positions: An Object-Oriented Model of Policy Networks, Bruce Bueno de Mesquita and Frans N. Stokman (eds), *European Community Decision Making: Models, Applications, and Comparisons*, New Haven: Yale University Press, 105-127

Stokman, Frans 2015
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